Math 5C Test 3 v2 – Fall 2022

Follow Instructions given on Canvas.

(1) Evaluate $\int_0^{\pi/2}\int_0^{\sqrt{x}}\int_0^{\sin x} \sqrt{x}\; dz\, dy\, dx$ *(7 points)*

$= \int_0^{\pi/2}\int_0^{\sqrt{x}} \sin x \sqrt{x}\, dy\, dx$

$= \int_0^{\pi/2} x \sin x \, dx \qquad u = x \quad dv = \sin x\, dx \qquad du = dx \quad v = -\cos x$

$= -x\cos x \Big]_0^{\pi/2} + \int \cos x\, dx$

$= -x\cos x + \sin x \Big]_0^{\pi/2}$

$= 1 - (0) = \boxed{1}$

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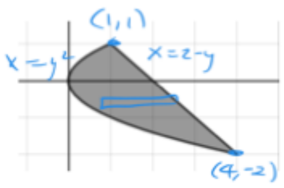
(2) Evaluate $\iint_D y\, dA$ where D is the region bounded by $x=y^2$ and $y = 2 - x$. *(10 points)*



$\int_{-2}^{1} \left((2-y) - (y^2)\right) dy$

$= 2y - \frac{1}{2}y^2 - \frac{1}{3}y^3 \Big]_{-2}^{1}$

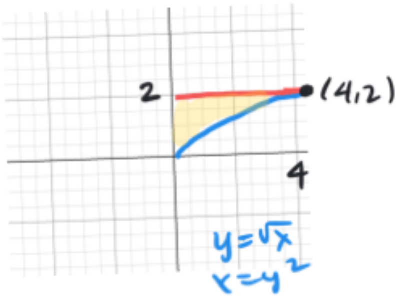
$= 2 - \frac{1}{2} - \frac{1}{3} - \left(-4 - 2 + \frac{8}{3}\right)$

$= 2 - \frac{1}{2} - \frac{1}{3} + 6 - \frac{8}{3}$

$= 8 - \frac{1}{2} - 3 = 5 - \frac{1}{2} = \boxed{\frac{9}{2}}$

(3) Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+4}\,dy\,dx$ You may want to reverse the order of integration.

(11 points)



$$= \int_0^2 \int_0^{y^2} \frac{1}{y^3+4}\,dx\,dy$$

$$= \int_0^2 \frac{y^2}{y^3+4}\,dy \qquad u=y^3+4 \quad du=3y^2dy$$

$$= \frac{1}{3}\int_4^{12} \frac{1}{u}\,du = \frac{1}{3}\ln|u|\Big]_4^{12}$$

$$= \frac{1}{3}(\ln 12 - \ln 4) = \boxed{\frac{1}{3}\ln 3}$$

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(4) Evaluate $\int_C xy^2\,ds$ where C is the line segment from (-3,0,1) to (4, 2, 5). *(11 points)*

$$x = -3+7t \qquad y = 2t \qquad z = 1+4t \qquad 0 \le t \le 1$$

$$ds = \sqrt{49+4+16}\,dt = \sqrt{69}\,dt$$

$$\int_0^1 (-3+7t)(4t^2)\sqrt{69}\,dt$$

$$= 4\sqrt{69}\int_0^1 (-3t^2+7t^3)\,dt = 4\sqrt{69}\left(-t^3+\frac{7}{4}t^4\right]_0^1$$

$$= 4\sqrt{69}\left(-1+\frac{7}{4}\right) = 4\sqrt{69}\left(\frac{3}{4}\right) = 3\sqrt{69}$$

(5) SET UP BUT DO NOT EVALUATE: integrals as specified to find the volume enclosed above the cone $z = \sqrt{\frac{1}{3}(x^2+y^2)}$ and inside the sphere $x^2 + y^2 + z^2 = 4$ in the first octant. In each part, sketch the necessary projection

(24points)

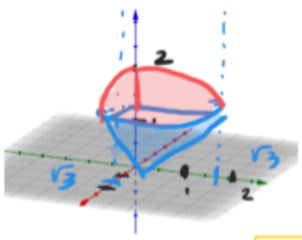
a) Sketch the solid

Intersection

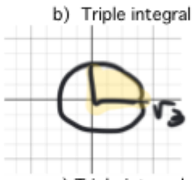
$4 - x^2 - y^2 = \frac{1}{3}(x^2+y^2)$

$4x^2 + 4y^2 = 12$

$x^2 + y^2 = 3, \ z = 1$

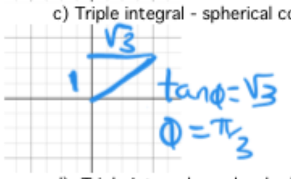


b) Triple integral - cylindrical coordinates.



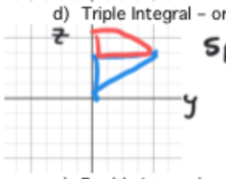
$$\int_0^{\pi/2}\int_0^{\sqrt{3}}\int_{r/\sqrt{3}}^{\sqrt{4-r^2}} dz\, r\, dr\, d\theta$$

c) Triple integral - spherical coordinates.



$$\int_0^{\pi/2}\int_0^{\pi/3}\int_0^{2} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

d) Triple Integral – order dx dz dy



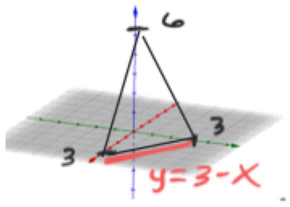
split

$$\int_0^{\sqrt{3}}\int_1^{\sqrt{4-y^2}}\int_0^{\sqrt{4-y^2-z^2}} dx\, dz\, dy + \int_0^{\sqrt{3}}\int_{y/\sqrt{3}}^{1}\int_0^{\sqrt{3z^2-y^2}} dx\, dz\, dy$$

e) Double integral- order dydx

$$\int_0^{\sqrt{3}}\int_0^{\sqrt{3-x^2}} \left(\sqrt{4-x^2-y^2} - \sqrt{\tfrac{1}{3}(x^2+y^2)}\right) dy\, dx$$

(6) Evaluate $\iint_S xz\, dS$ where S is the portion plane 2x+2y+z=6 in the first octant.



surface $z = 6-2x-2y$

$dS = \sqrt{z_x^2+z_y^2+1}\, dA$

$= \sqrt{4+4+1}\, dA = \sqrt{5}\, dA$

$\iint_S xz\, dS$

$= \int_0^3\int_0^{3-x} x(6-2x-2y)\sqrt{5}\, dA$

$= \sqrt{5}\int_0^3\int_0^{3-x} (6x-2x^2-2xy)\, dA$

$= \sqrt{5}\int_0^3 6xy-2x^2y-xy^2 \Big]_0^{3-x} dx$

$= \sqrt{5}\int_0^3 (6x(3-x)-2x^2(3-x)-x(3-x)^2)\, dx$

$= \sqrt{5}\int_0^3 (3x^3-12x^2+15x)\, dx$

$18x-6x^2+x^3$

$-6x^2+2x^3-3x$

$\sqrt{5}\left(\frac{3}{4}x^4-4x^3+\frac{15}{2}x^2\right)\Big]_0^3$

$\sqrt{5}\left(\frac{243}{4}-108+\frac{135}{2}\right) = \frac{82}{4}\sqrt{5} = \frac{41}{2}\sqrt{5}$

(7) Check all the boxes that apply. A function of three variables might appear in which of the following types of integrals? (6 points)

	Single Integral
	Double Integral
✓	Triple Integral
✓	Line Integral
✓	Surface Integral.

Domain must be in $\mathbb{R}^3$ – can be a

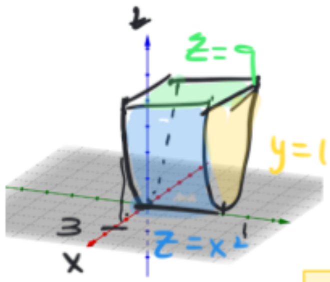
← solid

← curve

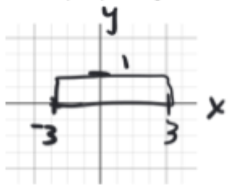
← surface

(8) SET UP ONLY :Find the volume of the solid bounded by the surface $z=x^2$ and the planes $y=0$, $y=1$, and $z=9$ according to the following directions. (sketch the solid). In each part, sketch the necessary projection

(20 points)

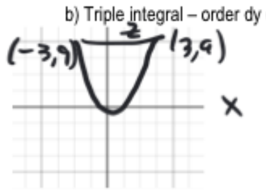


a) Triple integral – order dz dx dy



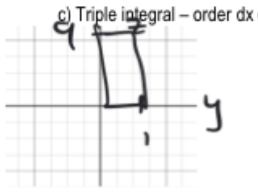
$$\int_0^1\int_{-3}^{3}\int_{x^2}^{9} dz\,dx\,dy$$

b) Triple integral – order dy dx dz



$$\int_0^9\int_{-\sqrt{z}}^{\sqrt{z}}\int_0^1 dy\,dx\,dz$$

c) Triple integral – order dx dy dz



$$\int_0^9\int_0^1\int_{-\sqrt{z}}^{\sqrt{z}} dx\,dy\,dz$$